

## NONLINEAR ANALYSIS OF GAS ABSORPTION IN TRAY COLUMNS USING KREMSEK EQUATION

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**ABSTRACT:** Greenhouse gas emission is one of the serious threats of air pollution to environment. Globally, the primary sources of greenhouse gas emissions are electricity and heat (31%), agriculture (11%), transportation (15%), forestry (6%) and manufacturing (12%). The main greenhouse gases are carbon dioxide, methane, nitrous oxide, hydrochlorofluorocarbons (HCFCs), hydrofluorocarbons (HFCs) and ozone in the lower atmosphere. Absorption is one of the effective unit operations for the purification or removal of gases from their mixture. Kremser equation is used for design and analysis of continuous tray absorption column. Nonlinear analysis is performed to simulate number of trays required for absorption at varied fraction of absorption ( $\alpha$ ) and absorption factor (A). The analysis concluded that good absorption takes place at  $0.4 < \alpha < 1$  and  $1 < A < 1.9$ .

**Keywords:** Absorption, Tray column, Kremser equation, Nonlinear analysis, Absorption factor

### I. INTRODUCTION:

Carbon dioxide emissions, primarily from the combustion of fossil fuels, have risen dramatically since the start of the industrial revolution [1]. Most of the world's greenhouse gas emissions come from a relatively small number of countries. Greenhouse gas intensity is a measure of the amount of emissions relative to GDP. It is highest in Russia and China with the United States below the world average. China, the United States, and the nations that make up the European Union are the three largest emitters on an absolute basis. Per capita greenhouse gas emissions are highest in the United States and Russia. Carbon dioxide emissions have risen rapidly for the past 70 years. However, they are projected to remain steady, albeit at a very high level, in the coming decades [2]. Emission reductions in developed economies are expected to offset carbon dioxide emission growth in developing countries [3].

Globally, the primary sources of greenhouse gas emissions are electricity and heat (31%), agriculture (11%), transportation (15%), forestry (6%) and manufacturing (12%). Energy production of all categories accounts for 72 percent of all emissions. Carbon dioxide accounts for about 76 percent of total greenhouse gas emissions [4]. Methane, primarily from agriculture, contributes 16 percent of greenhouse gas emissions and nitrous oxide, mostly from industry and agriculture, contributes 6 percent to global emissions [5]. The top 7 emitters (China, the US, the EU, India, Russia, Japan, and Brazil) are responsible for nearly two-thirds of global greenhouse gas emissions.

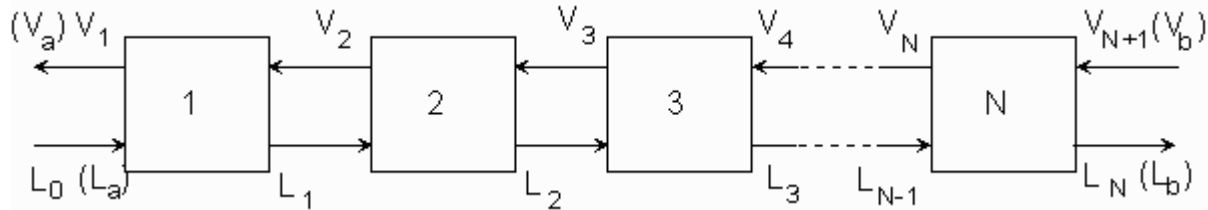
Gas absorption is a unit operation where a soluble vapor is absorbed from its mixture with an inert gas by means of a liquid in which the solute is more soluble [6]. Stripping is a process of removing gas dissolved in a liquid and where the liberated gas is taken up in a gas or vapor in contact with the liquid [7]. It is the reverse of gas absorption. The equipments for gas absorption are plate or tray columns and packed towers. The problems involving gas absorption are calculation of tower height, number of equilibrium stages, efficiency and vapour-liquid handling capacity of the equipment involved [8]. Absorption column is a plate column where gas is absorbed while a gas mixture bubbles through the liquid which passes down from plate to plate. Absorption factor is the product of the total pressure and the liquid flow rate in moles per second divided by the product of Henry's Law constant and the gas flow rate in moles per Second [9].

Kremser equation is the classical method for determining the number of stages, N, in counter-current mass exchange units, when both the operating line and the equilibrium line are linear [10]. Solving problems involving equilibrium stage separations requires simultaneous solution of the equilibrium and operating (component balance) expressions. Choice of a solution technique -- algebraic, graphical, or numerical -- depends on the form of the expressions.

Salvinder et al., (2019) presented an overview on the related efforts in terms of basic and advanced process control strategies are reviewed to provide further understanding on the key features that are required to optimize the operation of the absorption/stripping system [11]. In the present study, nonlinear analysis is performed to simulate number of trays required for absorption at varied fraction of absorption ( $\alpha$ ) and absorption factor (A).

**II. KREMSER EQUATION:**

Kremser Equation, or an "absorption factor method", provides an algebraic solution for analyzing equilibrium cascades. It cannot be used for every problem, but is convenient for dilute gas absorption



**Figure 1. Gas absorption system**

The equations will be developed for a countercurrent cascade of N stages. Begin by writing the steady state component balance over n-1 stages:

$$0 = L_0 x_0 - L_{n-1} x_{n-1} + V_n y_n - V_1 y_1$$

$$V_n y_n = L_{n-1} x_{n-1} + (V_1 y_1 - L_0 x_0)$$

The equilibrium expression will be written in terms of a "K-value":

$$y_n = K_n x_n$$

$$V_n y_n = V_n (K_n x_n)$$

The absorption factor is the ratio of the local slope of the operating curve to that of the equilibrium curve.

$$A_n = \frac{\left(\frac{L_n}{V_n}\right)}{K_n} = \frac{L_n}{V_n K_n}$$

$$V_n K_n = \frac{L_n}{A_n}$$

The absorption factor thus varies from stage to stage. These three expressions (component balance, equilibrium, absorption factor) are then combined and rearranged

$$V_n y_n = V_n K_n x_n = \frac{L_n x_n}{A_n} = L_{n-1} x_{n-1} + (V_1 y_1 - L_0 x_0)$$

$$L_n x_n = A_n (L_{n-1} x_{n-1}) + A_n (V_1 y_1 - L_0 x_0)$$

If the same steps were taken for a balance over n-2 and n-3 stages, the results would be:

$$L_{n-1} x_{n-1} = A_{n-1} (L_{n-2} x_{n-2}) + A_{n-1} (V_1 y_1 - L_0 x_0)$$

$$L_{n-2} x_{n-2} = A_{n-2} (L_{n-3} x_{n-3}) + A_{n-2} (V_1 y_1 - L_0 x_0)$$

These expressions are then "nested" into the first to obtain

$$\begin{aligned}
 L_n x_n &= A_n (A_{n-1} (L_{n-2} x_{n-2}) + A_{n-1} (V_1 y_1 - L_0 x_0)) + A_n (V_1 y_1 - L_0 x_0) \\
 &= A_n A_{n-1} (L_{n-2} x_{n-2}) + (A_n + A_n A_{n-1}) (V_1 y_1 - L_0 x_0) \\
 &= A_n A_{n-1} (A_{n-2} (L_{n-3} x_{n-3}) + A_{n-2} (V_1 y_1 - L_0 x_0)) + (A_n + A_n A_{n-1}) (V_1 y_1 - L_0 x_0) \\
 &= A_n A_{n-1} A_{n-2} (L_{n-3} x_{n-3}) + (A_n + A_n A_{n-1} + A_n A_{n-1} A_{n-2}) (V_1 y_1 - L_0 x_0)
 \end{aligned}$$

This process is repeated, until the balance over 1 stage is incorporated

$$\begin{aligned}
 L_1 x_1 &= A_1 (L_0 x_0) + A_1 (V_1 y_1 - L_0 x_0) \\
 &= A_1 (V_1 y_1)
 \end{aligned}$$

$$L_n x_n = (A_n A_{n-1} A_{n-2} \dots A_2 A_1) (V_1 y_1) + (A_n + A_n A_{n-1} + A_n A_{n-1} A_{n-2} + \dots + A_n A_{n-1} A_{n-2} \dots A_2) (V_1 y_1 - L_0 x_0)$$

The balance will be written one more time, over n stages

$$\begin{aligned}
 L_n x_n &= L_0 x_0 + V_{n+1} y_{n+1} - V_1 y_1 \\
 &= V_{n+1} y_{n+1} - (V_1 y_1 - L_0 x_0)
 \end{aligned}$$

Then the last two equations are set equal and rearranged:

$$\begin{aligned}
 V_{n+1} y_{n+1} - (V_1 y_1 - L_0 x_0) &= (A_n A_{n-1} A_{n-2} \dots A_1) (V_1 y_1) \\
 &\quad + (A_n + A_n A_{n-1} + A_n A_{n-1} A_{n-2} + \dots + A_n A_{n-1} A_{n-2} \dots A_2) (V_1 y_1 - L_0 x_0) \\
 &= (A_n + A_n A_{n-1} + A_n A_{n-1} A_{n-2} + \dots + A_n A_{n-1} A_{n-2} \dots A_1) (V_1 y_1) \\
 &\quad - (A_n + A_n A_{n-1} + A_n A_{n-1} A_{n-2} + \dots + A_n A_{n-1} A_{n-2} \dots A_2) (L_0 x_0)
 \end{aligned}$$

$$\begin{aligned}
 V_{n+1} y_{n+1} &= (A_n + A_n A_{n-1} + A_n A_{n-1} A_{n-2} + \dots + A_n A_{n-1} A_{n-2} \dots A_1 + 1) (V_1 y_1) \\
 &\quad - (A_n + A_n A_{n-1} + A_n A_{n-1} A_{n-2} + \dots + A_n A_{n-1} A_{n-2} \dots A_2 + 1) (L_0 x_0)
 \end{aligned}$$

If the absorbent is pure,  $x_0=0$ , and the second term vanishes. It is then convenient to define the fraction not absorbed, the ratio of solute leaving to solute fed

$$\frac{V_{n+1} y_{n+1}}{V_1 y_1} = \phi_A = \frac{1}{A_n + A_n A_{n-1} + A_n A_{n-1} A_{n-2} + \dots + A_n A_{n-1} A_{n-2} \dots A_1 + 1}$$

which can sometimes be used to compact the notation. This equation allows calculation of the recovery; but it is unlikely that anyone would have all the required absorption factors.

The absorption factor A varies from stage to stage as the liquid and vapor flows and equilibrium shift. The group method approximation says that we can assume an average, effective value of the absorption factor that is defined to be the same for all stages. Note, though, that if both the equilibrium curve and operating curve are straight lines, no approximation is involved. This allows algebraic simplification of the recovery fraction

$$\phi_A = \frac{1}{A_e + A_e^2 + A_e^3 + \dots + A_e^n + 1}$$

The rule for geometric series notifies that:

$$S_n = 1 + r + r^2 + \dots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

$$\phi_A = \frac{1 - A_e}{1 - A_e^{n+1}} = \frac{A_e - 1}{A_e^{n+1} - 1}$$

A similar simplification can be done on the  $L_0 x_0$  term, noting that the order is one less. The full equation is thus

$$\begin{aligned}
 V_{n+1}y_{n+1} &= V_1y_1 \left( \frac{A_e^{n+1} - 1}{A_e - 1} \right) - L_0x_0 \left( \frac{A_e^n - 1}{A_e - 1} \right) \\
 &= \frac{V_1y_1}{\phi_A} - L_0x_0 \left( \frac{A_e^n - 1}{A_e - 1} \right)
 \end{aligned}$$

The coefficient on  $L_0x_0$  represents the consequences of both impure absorbent and the fact that vapor flow may do some stripping of the enriched absorbent. It thus makes sense to express this quantity in terms of the stripping factor:

$$S_e = \frac{1}{A_e} \quad \phi_S = \frac{S_e - 1}{S_e^{n+1} - 1}$$

Beginning by setting up a common denominator, the  $L_0x_0$  coefficient can be rewritten to obtain

$$\begin{aligned}
 \left( \frac{A_e^n - 1}{A_e - 1} \right) \left( \frac{\phi_A}{\phi_A} \right) &= \frac{1}{\phi_A} \left( \frac{A_e^n - 1}{A_e - 1} \right) \left( \frac{A_e - 1}{A_e^{n+1} - 1} \right) = \frac{1}{\phi_A} \left( \frac{A_e^n - 1}{A_e^{n+1} - 1} \right) \\
 &= \frac{1}{\phi_A} \left( \frac{\frac{1}{S_e^n} - 1}{\frac{1}{S_e^{n+1}} - 1} \right) = \frac{1}{\phi_A} \left( \frac{S_e^{n+1} - S_e}{S_e^{n+1} - 1} \right) = \frac{1}{\phi_A} \left( \frac{S_e^{n+1} - 1 + 1 - S_e}{S_e^{n+1} - 1} \right) \\
 &= \frac{1}{\phi_A} \left( 1 + \frac{1 - S_e}{S_e^{n+1} - 1} \right) = \frac{1}{\phi_A} (1 - \phi_S)
 \end{aligned}$$

so that the overall equation is

$$V_{n+1}y_{n+1} = \frac{V_1y_1}{\phi_A} - \frac{L_0x_0(1 - \phi_S)}{\phi_A}$$

This equation is useful in solving some problems.

It is assumed that the flow rates L and V and the equilibrium K-value are constants. This means that both the equilibrium and operating curves will be straight lines and that the absorption and stripping factors are constants.

$$\begin{aligned}
 V_{n+1}y_{n+1} &= V_1y_1 \left( \frac{A_e^{n+1} - 1}{A_e - 1} \right) - L_0x_0 \left( \frac{A_e^n - 1}{A_e - 1} \right) \\
 y_{n+1} &= \left( \frac{V_1}{V_{n+1}} \right) \left( \frac{A_e^{n+1} - 1}{A_e - 1} \right) y_1 - \left( \frac{L_0}{V_{n+1}} \right) \left( \frac{A_e^n - 1}{A_e - 1} \right) x_0 \\
 y_{n+1} &= \frac{A_e^{n+1} - 1}{A - 1} y_1 - \frac{L}{V} \left( \frac{A^n - 1}{A - 1} \right) x_0
 \end{aligned}$$

Define the hypothetical equilibrium vapor composition, substitute, and rearrange.

$$y_0^* = Kx_0$$

$$x_0 = \frac{y_0^*}{K}$$

$$\begin{aligned} y_{n+1} &= \frac{A^{n+1} - 1}{A - 1} y_1 - \frac{L}{VK} \left( \frac{A^n - 1}{A - 1} \right) y_0^* \\ &= \frac{A^{n+1} - 1}{A - 1} y_1 - A \left( \frac{A^n - 1}{A - 1} \right) y_0^* \end{aligned}$$

$$y_{n+1}(A - 1) = (A^{n+1} - 1)y_1 - A(A^n - 1)y_0^*$$

$$Ay_{n+1} - y_{n+1} = A^{n+1}y_1 - y_1 - A^{n+1}y_0^* + Ay_0^*$$

$$y_1 - y_{n+1} = A^{n+1}(y_1 - y_0^*) + A(y_0^* - y_{n+1})$$

Next, a rearranged version of the balance over n stages gives,

$$V_{n+1}y_{n+1} = Vy_1 + L_n x_n - L_0 x_0$$

$$y_{n+1} = y_1 + \frac{L}{V} x_n - \frac{L}{V} x_0$$

$$y_{n+1} = y_1 + (AK)x_n - \frac{L}{V} \frac{y_0^*}{K}$$

$$y_{n+1} = y_1 + Ay_n^* - Ay_0^*$$

$$y_{n+1} - y_1 = A(y_n^* - y_0^*)$$

Note that  $y_n^* = y_n$ . This can be used to calculate A from known endpoint compositions

$$A = \frac{y_{n+1} - y_1}{y_n - y_0^*}$$

The last two equations are combined to get

$$y_1 - y_{n+1} = A(y_0^* - y_n^*) = A^{n+1}(y_1 - y_0^*) + A(y_0^* - y_{n+1})$$

$$A(y_0^* - y_n^* - y_0^* + y_{n+1}) = A^{n+1}(y_1 - y_0^*)$$

$$(y_{n+1} - y_n^*) = A^n(y_1 - y_0^*)$$

$$A^n = \frac{(y_{n+1} - y_n^*)}{(y_1 - y_0^*)}$$

$$n \ln(A) = \ln \left( \frac{y_{n+1} - y_n^*}{y_1 - y_0^*} \right)$$

$$n = \frac{\ln \left( \frac{y_{n+1} - y_n^*}{y_1 - y_0^*} \right)}{\ln(A)} = \frac{\ln \left( \frac{y_{n+1} - y_n^*}{y_1 - y_0^*} \right)}{\ln \left( \frac{y_1 - y_{n+1}}{y_0^* - y_n^*} \right)}$$

In other terms,

$$\alpha = \frac{y_{n+1} - y_1}{y_{n+1} - y_0^*} = \frac{A^{n+1} - A}{A^{n+1} - 1}$$

which can be used to determine the number of stages needed to make a separation by absorption.

III. NONLINEAR ANALYSIS:

The analysis was performed by assuming the values of  $\alpha$  from 0 to 1 and A greater than 1. In the rearranged form of Kremser equation given below,

$$n = \frac{\ln\left(\frac{1-\alpha}{1-\alpha A}\right)}{\ln A}$$

if A is 1, then denominator becomes zero and the number of plates becomes infinity. If  $\alpha$  is 1,  $\ln(\infty)$  becomes indeterminate. Hence,  $\alpha$  and A were studied in the ranges of  $0 < \alpha < 1$  and  $1 < A < 1.9$ . When  $A > 1.9$ , number of plates becomes zero. Table 1 shows number of plates calculated at different values of  $\alpha$  and A.

Table 1. Number of theoretical trays at different values of  $\alpha$  and A

A	$\alpha=0$	$\alpha=0.1$	$\alpha=0.2$	$\alpha=0.3$	$\alpha=0.4$	$\alpha=0.5$	$\alpha=0.6$	$\alpha=0.7$	$\alpha=0.8$	$\alpha=0.9$
1.05	0	0	0	0	1	1	1	2	4	7
1.1	0	0	0	0	1	1	1	2	3	6
1.15	0	0	0	0	1	1	1	2	3	6
1.2	0	0	0	0	1	1	1	2	3	5
1.25	0	0	0	0	1	1	1	2	3	5
1.3	0	0	0	0	1	1	1	2	2	4
1.35	0	0	0	0	1	1	1	2	2	4
1.4	0	0	0	0	1	1	1	2	2	4
1.45	0	0	0	0	1	1	1	1	2	4
1.5	0	0	0	0	0	1	1	1	2	3
1.55	0	0	0	0	0	1	1	1	2	3
1.6	0	0	0	0	0	1	1	1	2	3
1.65	0	0	0	0	0	1	1	1	2	3
1.7	0	0	0	0	0	1	1	1	2	3
1.75	0	0	0	0	0	1	1	1	2	3
1.8	0	0	0	0	0	1	1	1	2	3
1.85	0	0	0	0	0	1	1	1	2	3
1.9	0	0	0	0	0	1	1	1	2	3
1.95	0	0	0	0	0	0	0	0	0	0

Since the number of plates is not changed for  $\alpha < 0.4$ , graph between number of plates and absorption factor is plotted at  $0.4 < \alpha < 1$ . Figure 1 shows the plot of number of theoretical plates as a function of absorption factor at different values of  $\alpha$ .

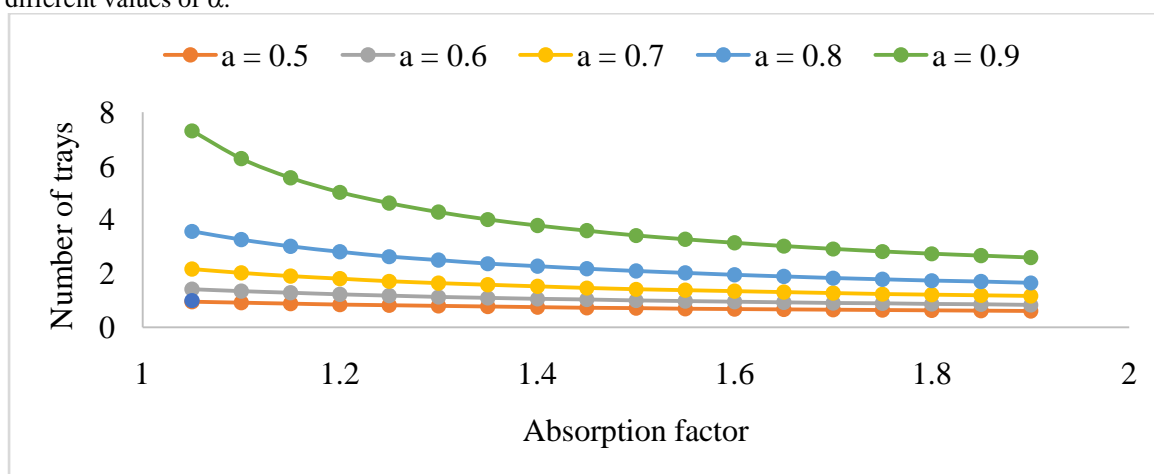


Figure 1. Number of theoretical plates as a function of absorption factor at different values of  $\alpha$  (a). From Figure 1, it is concluded that good absorption takes place at  $0.4 < \alpha < 1$  and  $1 < A < 1.9$ .

#### **IV. CONCLUSION:**

The present study aimed to perform nonlinear analysis to simulate number of trays required for absorption at varied fraction of absorption ( $\alpha$ ) and absorption factor (A). Kremser equation is used for analysis of continuous tray absorption column. Nonlinear analysis is performed to simulate number of trays required for absorption at varied fraction of absorption ( $\alpha$ ) and absorption factor (A). The analysis concluded that good absorption takes place at  $0.4 < \alpha < 1$  and  $1 < A < 1.9$ .

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