American Journal of Multidisciplinary Research & Development (AJMRD)
Volume 03, Issue 07 (July- 2021), PP 40-44
ISSN: 2360-821X
www.ajmrd.com

Research Paper **Open Access**

Anti-domination Number of a Graph

MallikarjunBasannaKattimani
Department of Mathematics, The Oxford College of Engineering,
Bangalore-560 068, Karnataka State-INDIA,
Corresponding Author: MallikarjunBasannaKattimani

Abstract: Let $G=(V,E)$ be a graph, a set $D\subseteq V$ is a $dominating\ \ set$ of G, if every vertex in $V-D$ is adjacent to at least one vertex in D. The $domination\ \ number\ \gamma(G)$ of G is the minimum cardinality of a dominating set [1] and [6].

We define, Let D be a minimum dominating set of G. A set K of vertices in $V-D$ of G is an $Anti\ dominating\ \ set$ with respect to D, if every vertex in D is adjacent to at least one vertex in K. The $Anti\ domination\ \ number\ \gamma_k(G)$ of G is the minimum cardinality of an $Anti\ dominating\ \ set$ is well defined. In this paper we obtained exact values of $\gamma_k(G)$ of G for some standard graphs and also we establish some general results and Nordhus and Guddamm type result on this new parameter.

Key words: Dominating set, Anti-dominating set.

Subject Classification: 05C69

I. Introduction

The Graph considered here are finite, connected, nontrivial, undirected, without loops or multiple edges. Any undefined term in this paper may be found in Harary [4].

A vertex in a graph $G=(V,E)$ is said to be dominate every vertex adjacent to it. A set D of vertices in G is a $dominating\ \ set$, if every vertex in $V-D$ is dominated by at least one vertex in D. Dominating sets were defined by Berge [1] (Where they are called externally stable sets) and Ore [6].

The $domination\ \ number\ \gamma(G)$ of a graph G is the smallest number of vertices in any minimal dominating set. It appears in various puzzle questions. In a regular chessboard and the five chess pieces: Rook, Bishop. Knight, King and Queen all these must tour the board using only legal moves, landing on every square exactly once. One instance is the so called Five queens problem on the chessboard: It is required to place five queens on the board in such positions that they dominate each square as shown in (fig.1), no smaller number of queens will suffice, so that $\gamma(G)=5$. In 1850's five is the minimum number of queens that can dominate all of the squares of 8×8 chessboard. The five queen's Problem is to find a dominating set of five queens, [1] and [6].

					Q		
		Q					
				Q			
						Q	
			Q				

(a)

	Q						
			Q				
				Q			
					Q		
							Q

(b)

Figure 1.

Among the many solutions to this problem, the two in Fig.1 are particularly interesting. In the first solution (Fig.1a) no queen is dominated by any other queen, while in the second solution (fig.1b) the opposite is essentially true, every queen is dominated by at least one other queen. The second solution suggests the following definition: A set T of vertices in G is a *total dominating set*, if every vertex in V is dominated by at least one vertex in T. The *total dominating sets* were first defined and studied by Cockayne, Dawes and Hedetniemi [3].

Connecting to the above five-queens problem, five queens are placed in such places in a chessboard as shown in fig.1 that all remaining 59 squares are attacked or occupied by a queen. Hence, every square is dominated by at least one of the five queens. A set of five queens is called a *dominating set*.

But in fig.1 the queen is placed in such a way that it poses threat for all 59 squares, hence it can rightly be termed as *dominating set*, so the problem arises in front of us is How to safeguard the 59 squares? by five queens. Contrary to this in fig.1 the rook is placed in particular places out of 59 squares before the queen placed, in such a way that Rook poses threat to queen, hence it can rightly be termed as *Anti dominating set*, and denoted by K.

Then, our solution is, before occupying five particular places by a queen (fig.1), we place a smallest possible number than five in different particular places out of 59 squares by rooks, which poses threat to the queens. Having this done we can protect all 59 squares as shown in (fig.2a) or (fig.3a) with the initial arrangement of rooks in a particular position (fig.2a) or (fig.3a). Hence we say that every queen is dominated by at least one of the three rooks in (fig.2a) or (fig.3a). We call it as *Anti dominating set* over the chessboard.

Similarly, we can use bishop (fig.2b) and (Fig.3b) or knight in (fig. 2c) and (Fig. 3c) and knight instead of rook as mentioned in the following

				R	Q		
		Q					
				Q		R	
						Q	
		R	Q				

5-Q-queen
3-R-rook
(a)

					Q	B	
		Q					
				Q			
		B				Q	
			Q		B		

5-Q-queen
3-B-bishop
(b)

					Q		
		Q				K	K
				Q			
	K					Q	
			Q				

5-Q-queen
3-K-knight
(c)

Figure 2

	Q						R
			Q		R		
				Q			R
					Q		
							Q

5-Q-queen
3-R-rook
(a)

	Q						
		B					
			Q		B		
				Q			
					Q		
						B	
							Q

5-Q-queen
3-B-bishop
(b)

	Q						
		K	Q				
				Q			
		K			Q	K	
			K				
							Q

5-Q-queen
4-K-knight
(c)

Figure 3

In our discussion, three is the minimum number of rooks that can dominate all of the five queens of 8×8 chessboard. The three-rook problem is to find an $Anti\ dominating\ \ set$. Hence it motivates.

We define, let D be minimum $dominating\ \ set$ of G. A set K of vertices in $V-D$ of G is an $Anti\ dominating\ \ set$ with respect to D, if every vertex in D is adjacent to at least one vertex in K. The $Anti\ domination\ \ number\ \gamma_k(G)$ of G is the minimum cardinality of an $Anti\ dominating\ \ set$.

Results

Exact values of $\gamma_k(G)$ for some standard graph are given in Theorem 1.

Theorem 1.

(i) $\gamma_k(C_p) = \left\lfloor \frac{p}{3} \right\rfloor$ (1)

(ii) $\gamma_k(P_p) = \left\lfloor \frac{p}{3} \right\rfloor$ (2)

(iii) $\gamma_k(K_p) = 1$ (3)

(iv) $\gamma_k(K_{m,n}) = \begin{cases} 1 & \text{if } m,n \le 2 \\ 2 & \text{if } m,n \ge 3 \end{cases}$ (4)

(v) $\gamma_k(K_{1,n}) = 1$ (5)

(vi) $\gamma_k(W_p) = 1$ (6)

Where, $\lfloor x \rfloor$ is the greatest positive integer not greater than x

Theorem 2. For any graph G, $\gamma_k(G) \le \gamma(G)$ (7)

Proof. (7) follows from the definition of $\gamma(G)$ and $\gamma_{Ad}(G)$

Theorem 3. For any graph G, $\gamma_k(G) \le \gamma_t(G)$ (8)

Proof. Since $\gamma(G) \le \gamma_t(G)$ and from (7), we have $\gamma_k(G) \le \gamma_t(G)$. Hence the result.

Theorem 4. For any graph G, $\gamma_k(G) \le p - \beta_0$ (9)

Proof. We know that any graph $\gamma(G) \le \alpha_0$ and also $\alpha_0 + \beta_0 = p$ therefore $\gamma_k \le p - \beta_0$ and from (7), we get $\gamma_k(G) \le p - \beta_0$. Hence the result.

The following results are strait forward.

Theorem 5. If G is a K_p or W_p or $K_{1,n}$, then $\gamma_k(G) = \gamma(G)$

Theorem 6. For any graph G, $u \in D$ and $N(u) = v$ is a pendent vertex, then $\gamma_k(G) = c(T)$, where $c(T)$ is a cut vertex.

Theorem 7. If G is connected and $\Delta(G) < p - 1$ then $\gamma_k(G) \le p - \Delta(G)$.

Proof. Since, we know that $\gamma_t(G) \le P - \Delta(G)$, [2] and from (9), we get $\gamma_k(G) \le p - \Delta(G)$.

Theorem 8. Let T be a tree such that every cut vertex is adjacent to at least two end vertices. Then $\gamma_k(T) = \gamma(G)$.

Proof. Since, every cut vertex is belongs to D and $N(D) = v$ is an end vertex, therefore $\gamma_k(T) = \gamma(G)$. Hence the result.

Theorem 9. Let G be a graph with p vertices, q edges and maximum degree Δ, then $p - q \leq \gamma_k(G) \leq p - \Delta$.

Proof. We know that $\gamma(G) \leq p - \Delta$, [2]. And from (7) upper bound holds true. Clearly G is connected, we have $p \leq q + 1$ and $\gamma_k(G) \geq 1$. Hence the result.

Theorem 10. Let $K_{m,n}$ be a complete bipartite graph. Then $\gamma_k(K_{m,n}) = \gamma(K_{m,n}) = 2$.

Proof. Let $K_{m,n}$ be a complete bipartite graph on vertex sets V_1 and V_2 such that $|V_1| = m$ and $|V_2| = n$. Let D be a minimum dominating set in G. Suppose $v_i \in V_1$. Then v_i is adjacent with at least n vertices of V_2. And v_j is adjacent with at least m vertices of V_1. Thus $v_i, v_j \in D$. Hence $\gamma(K_{m,n}) = 2$. Then v_i is dominated by one vertex of V_2 and v_j is dominated by one vertex of V_1. Thus $\gamma_k(K_{m,n}) = 2$. Hence the result.

Reference

[1] C. Berge; Graph and Hypergraphs, North Holland, Amsterdam, 1973.

[2] E. J. Cockayne and S. T. Hedetniemi; Towards a theory of domination in graphs, *Networks*, 7, 247-261 (1977).

[3] E. J.Cockayne, R. M. Dawes and S. T. Hedetniemi; Total domination in graphs *Networks*,10, 211-215 (1980).

[4] F. Harary; *Graph theory*, Addison-Wesley, Reading, Mass. (1969).

[5] J. W.Glaisher, On the problem of Eight Queens, The Phil. Mag., Series 4, 457.[216].

[6] O. Ore; Theory of Graphs, Am Soc. Colloq. Publ., 38, Providence, R1, 1962.